Chapter 2: Measurement Techniques

Errors, Accuracy, Precision, and Propagation of Uncertainty

Types of Errors

Systematic Errors

Errors directly attributed to equipment. These **can** generally be mitigated. Causes *consistent* variation.

1. Instrument is malfunctioning.
2. Instrument is incorrectly calibrated.
3. Instrument is incorrectly used.
4. Instrument degradation due to wear and tear and use.

Causes readings to differ from the true value by a consistent amount each time a reading is made.

Random Errors

Errors caused by unknown and unpredictable changes in the experiment or experimental conditions. These **cannot** be eliminated. Causes *chaotic* variation.

1. Weather conditions.
2. Laboratory conditions.
3. Anything else that likely cannot be controlled.

Cause readings to vary around the mean value in an unpredictable way from one reading to another.

Types of Errors (Examples)

Systematic Errors

1. A force meter becoming weaker with time.
2. A magnet with weakened polarity.
3. Improperly calibrated thermometer.
4. Measuring volume in a graduated cylinder at an angle.
5. Errors in measuring solar radiation because tool is placed in the shade.
6. Poor contact between detector and surface.

Random Errors

1. Circulating air causing changes to thermal readings.
2. Electrical noise in a circuit.
3. Subtle changes in posture changing the value of your weight.
4. Variations in sunlight intensity due to change in cloud cover.

Zero Errors

When an instrument gives a non-zero reading when the true value of the quantity is zero.

Examples:

1. An ammeter measures a current in a vacuum of 0.001 A.
2. Measuring distance with a ruler and not aligning it to zero.

Accuracy and Precision

Accuracy

When results and/or data are close to the true value.

A quantity is accurate if the measured quantity is close to the value of the true quantity.

Example:

1. The acceleration of free-fall is 9.81 ms^-2. A group of experimenters calculated the acceleration of free-fall to be 9.808.

This is an ACCURATE result!

Precision

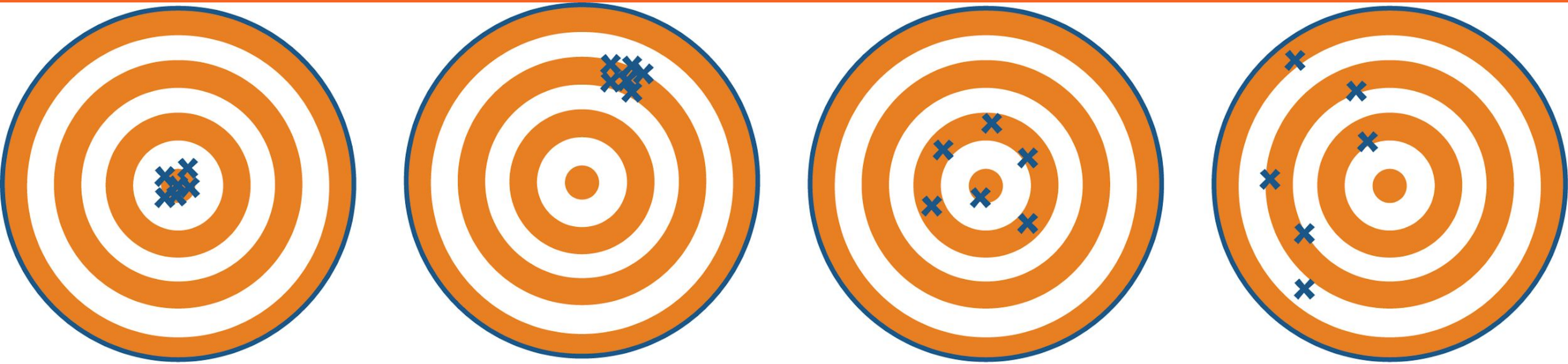
The smallest change in value that can be measured by an instrument or operator.

A measurement is precise if several measurements give the same, similar values without much variation about the mean value.

Example:

1. Four different teams measure the length of some object and get the following results: 1.1 m, 1.09 m, 1.11 m, and 1.12 m

This is a PRECISE result!



High Accuracy
High Precision

Low Accuracy
High Precision

High Accuracy
Low Precision

Low Accuracy
Low Precision

Accuracy and Precision

Results can be accurate and/or precise and they can also be neither accurate nor precise!

Experimental data should strive for both.

Errors in experimental data affect the accuracy and precision of data.

Minimizing Error

1. Taking more data measurements generally can mitigate error.
   a. In statistics, more data measurements and samples brings results closer to statistical averages.
2. Proper calibration of equipment.
3. Consistency in experimental procedure.

Cambridge commonly asks how error can be minimized or results can be improved. They also tend to ask where limitations can generate error. ALWAYS suggest ideas within the framework of the experiment and **BE SPECIFIC**!

Example of Limitation and Improvements

Problem

1. Two results are not valid enough to draw a conclusion.
2. The ruler is too wide to measure the depth of a crater.
3. There may be parallax error when measuring the top of the crater.
4. It is difficult to release the ball consistently without variation.
5. The crater lip is of varying height.

Improvement

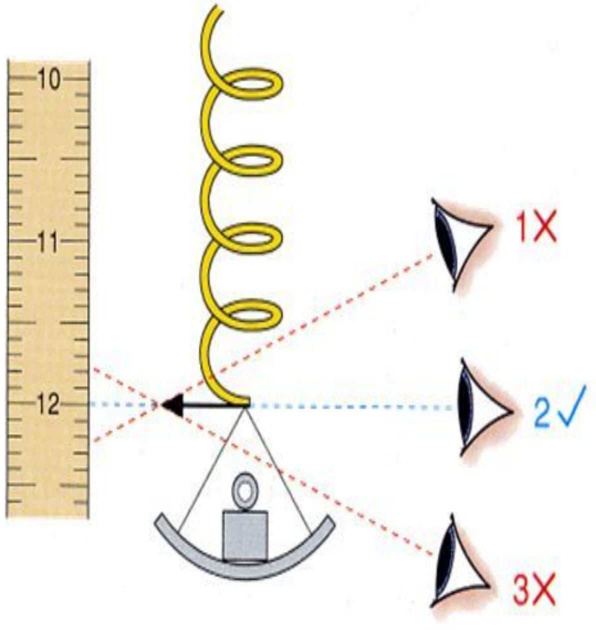
1. Take more results.
2. Use a needle or instrument to make marks and measure the marks.
3. Keep eyes parallel to the horizontal level of the sand.
4. Use an electromagnet to release the ball.
5. Always take your measurements consistently and from the same position.

Parallax Error

A change of an object's length or location based on perspective.

The perspective comes a result of looking at an object for different angles.

This is solved by always looking level on or perpendicular to the measuring tool.



Propagation of Uncertainty

Uncertainty is defined as “the estimate in a reading of the difference between the reading and true value of the quantity being measured.

The tendency that the total or final uncertainty in a result is greater than the uncertainty of the individual measurements.

There are three types of uncertainty.

1. Absolute Uncertainty
2. Fractional Uncertainty
3. Percentage Uncertainty

Absolute Uncertainty

Absolute Uncertainty refers to the uncertainty **expressed in the same units as the measurement.**

Example: The length of a computer disk label might read (5.5 +/- 0.1) cm. This would be read as 5.5 cm plus or minus 0.1 cm.

The absolute uncertainty is the +/- 0.1 cm.

Note that absolute uncertainty has units.

Fractional Uncertainty

Fractional Uncertainty is the **ratio of the of the uncertainty over the measurement.**

Example: The length of a computer disk label might read (5.5 +/- 0.1) cm. This would be read as 5.5 cm plus or minus 0.1 cm.

5.5 +/- (0.1 cm/5.5 cm) = 5.5 cm +/- 0.018

0.018 is the fractional uncertainty. Note that fractional uncertainty has no units due to prior cancellation.

Percentage Uncertainty

Percentage Uncertainty is the **fractional uncertainty multiplied by 100.**

Example: The length of a computer disk label might read (5.5 +/- 0.1) cm. This would be read as 5.5 cm plus or minus 0.1 cm.

5.5 +/- (0.1 cm/5.5 cm) = 5.5 cm +/- *0.018*

0.018 is the fractional uncertainty.

0.018 x 100 = 1.8% ; 1.8% is the percentage uncertainty. Note that it is a percentage!

Therefore our uncertainty would read as 5.5 cm +/- 1.8%!

Like fractional uncertainty, percentage uncertainty has no units!

Propagation of Uncertainty in Calculations

Addition and Subtraction

Absolute uncertainties are added together regardless of operation to obtain the absolute uncertainty in the result.

Example: Change in Temperature

(99.2 C +/- 1.5 C) - (27.6 C +/- 1.5 C) =

71.6 +/- 3.0 C

(3.0 / 71.6) = 0.0418; (0.418 x 100) = 4.2%

So 71.6 +/- 4.2 % is the answer.

Multiplication and Division

Percent uncertainties are added together regardless of the operation to obtain the percentage uncertainty.

Example: Area of a Plate

(5.5 cm +/- 1.8%) x (6.4 cm +/- 1.6%) = 35 cm^2 +/- 3.4%

3.4% is 0.034 as fractional uncertainty.

0.034 x 35 cm = 1.19 cm so 35 +/- 1.0 cm^2 is the answer.

Propagation of Uncertainty in Calculations

Expressions with Powers:

If a measurement is taken to a power, the percent uncertainty is multiplied by that power to obtain the percent uncertainty in the result.

Example: Volume of a Sphere

r = 6.20 cm +/- 2.0 %

V=(4/3) π r^3 = (4/3) π $(6.20 \text{ cm} +/- 2.0)^3$ = 998 cm^3 +/- 6.0 %

6.0 % / 100 = 0.06

0.06 x 998 = 60 so the final answer is 998 cm^3 +/- 60 cm^3

Example Problem 1

The diameter of a tire is reported as 55.0 cm +/- 0.5 cm and the inner diameter of a tire is reported as 21.0 cm +/- 0.7 cm.

Determine the absolute, fractional, and percentage uncertainty of the sum of the diameters.

55.0 cm + 21.0 cm = 76.0 cm

Total Absolute Uncertainty: 0.5 cm + 0.7 cm = 1.2 cm.

Recall that when adding or subtracting, we *ADD* the absolute uncertainties!

Total Fractional Uncertainty: 1.2/76.0 = 0.016.

Total Percentage Uncertainty: 0.016 x 100 = 1.6%

Example Problem 2

The speed of a car is reported as 10.0 ms^{-1} +/- 0.12 ms^{-1}. The amount of time it takes to clear a certain distance D is reported as 5.00 s +/- 0.05 s.

Calculate the distance travelled in this time interval and the absolute uncertainty in the distance.

Using d = st we see that the uncertainties must be converted to percentage uncertainties since we are involving multiplication and then add them together.

Fractional Uncertainty in Speed = (0.12/10.0) = 0.012 while the Percentage Uncertainty will be 0.012 x 100 hence 1.2%. The fractional uncertainty in the time will be (0.05/5.00) = 0.01 while the Percentage Uncertainty will be 1%. Hence the total percentage uncertainty is 2.2%

Solving for d we get 50 m but we now need the absolute uncertainty. Converting the total percentage uncertainty to fractional uncertainty we obtain 0.022. Then we multiply this by the measurement to get the absolute uncertainty. Hence we get 50 m +/- 1.1 m.

Uncertainty Flow Chart

Uncertainties

AU ⟷ FU ⟷ PU

Where:
AU = absolute uncertainty
FU = fractional uncertainty
PU = percentage uncertainty

To convert from AU to FU, FU = AU / measurement.
To convert from FU to PU, PU = FU x 100.

To convert in reverse, PU to FU, FU = PU / 100.

To convert in reverse, FU to AU, FU x measurement = AU.

Example Problem 3

Calculate the total percentage uncertainty in the volume of a cube with length = 10.0 cm + 0.01 cm.

We know that volume of a cube is equal to $V = L^3$.

Hence we need to convert the absolute uncertainty to percentage uncertainty and then apply the rules for exponents.

Thus doing the conversion we obtain 1000 cm^3 +/- 0.3%. The 0.3% is obtained from (0.01/10.0) x 100 = 0.1% and then considering this uncertainty of 0.1 is impacted three times from the exponent hence 3(0.1) which is 0.3%.